B. Tech.

(SEM. V) ODD SEMESTER THEORY EXAMINATION, 2016-17
STRUCTURAL ANALYSIS-II

Time : 3 Hours **Max. Marks : 100**

SECTION – A

Note : Attempt **all** the questions.

1. Attempt **all** questions. All carries equal marks. **(10 × 2 = 20)**

a. **Distinguish between flexibility method and stiffness method.**

Ans. Refer Q. 4.3, 2 Marks Questions, Page SQ–12B, Unit-4.

b. **Define shape factor.**

Ans. Refer Q. 5.4, 2 Marks Questions, Page SQ–15B, Unit-5.

c. **How we determine the horizontal thrust for two hinged arch ?**

Ans. Refer Q. 2.5, 2 Marks Questions, Page SQ–6B, Unit-2.

d. **Define distribution factor.**

Ans. Refer Q. 1.11, 2 Marks Questions, Page SQ–2B, Unit-1.

e. **Derive the shape factor of rectangle.**

Ans. Refer Q. 5.3, Page 5–3B, Unit-5.

f. **Write any two methods to analysis continuous beam with its equation.**

Ans. Refer Q. 1.17, 2 Marks Questions, Page SQ–4B, Unit-1.

g. **Explain Muller Breslau's principle.**

Ans. Refer Q. 2.1, 2 Marks Questions, Page SQ–5B, Unit-2.

h. **What is relative stiffness ? Write relative stiffness of continuous beam.**

Ans. **Relative Stiffness :** Refer Q. 1.7, 2 Marks Questions, Page SQ–2B, Unit-1.

Relative Stiffness of Continuous Beam : Refer Q. 1.8, 2 Marks Questions, Page SQ–2B, Unit-1.

i. **What is the maximum tension and minimum tension on a cable of suspension bridge ?**

Ans. Refer Q. 3.4, 2 Marks Questions, Page SQ–9B, Unit-3.

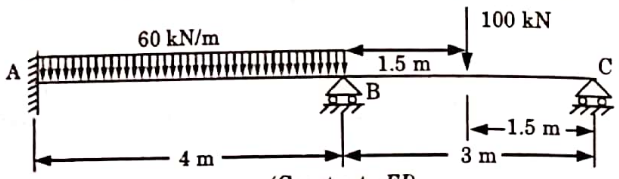
j. **What is plastic hinge ?**

Ans. Refer Q. 5.3, 2 Marks Questions, Page SQ–15B, Unit-5.

SECTION-B

2. Attempt any five questions. (5 × 10 = 50)

a. **Analyze the following continuous beam Fig. 1 using the flexibility method of matrix analysis. Draw BMD.**



(Constant : *EI*)

Fig. 1.

Ans. Refer Q. 4.3, Page 4–6B, Unit-4.

b. **Draw the bending moment diagram for the continuous beam shown in Fig. 2 using moment distribution method. *EI* is constant.**

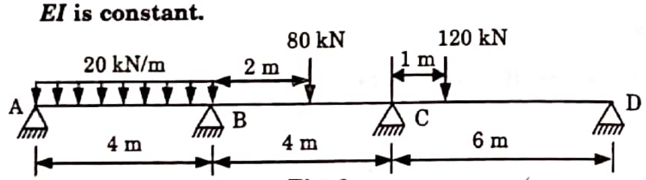


Fig. 2.

Ans. Refer Q. 1.15, Page 1–29B, Unit-1.

c. **A foot bridge is carried over a river of span 90 m. The supports are 3 m and 12 m higher than the lowest point of the cable. Determine the length of the cable. If the horizontal deck is loaded by UDL of 20 kN/m, find the tension in the cable.**

Ans. Refer Q. 3(b), Page SP–8B, Solved Paper 2014-15.

d. **Draw the schematic diagrams for horizontal thrust, bending moment at any section, radial shear and normal thrust at any given section for a typical two hinged symmetrical parabolic arch.**

Ans. Refer Q. 2.11, Page 2–24B, Unit-2.

e. **Define shape factor and obtain its value for T-section with the following dimension shown in the Fig. 3. If the yield stress is 250 N/mm². Find M_p.**

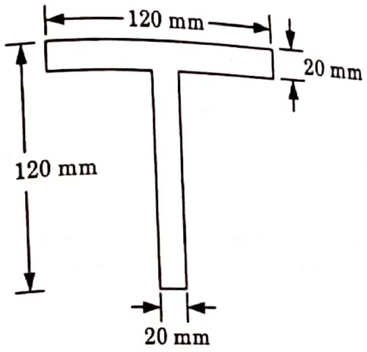


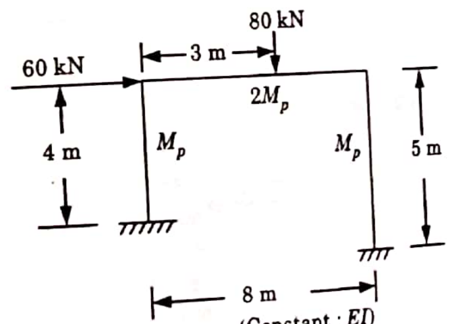
Fig. 3.

Ans. **Shape Factor :** Refer Q. 5.3, Page 5–3B, Unit-5.
Numerical : Refer Q. 5.6, Page 5–9B, Unit-5.

f. **Derive the influence line diagram for reactions and bending moment at any section of a simply supported beam. Using the ILD, determine the support reactions and find bending moment at 2 m, 4 m and 6 m for a simply supported beam of span 8 m subjected to three point loads of 10 kN, 15 kN and 5 kN placed at 1 m, 4.5 m and 6.5 m respectively.**

Ans. Refer Q. 2.3, Page 2–5B, Unit-2.

g. **Determine the plastic moment capacity M_p for the frame shown in Fig. 4 given below.**



(Constant : *EI*)

Fig. 4.

Ans. Refer Q. 5.16, Page 5–24B, Unit-5.

Section-C

Attempt any **two** questions. (2 × 15 = 30)

3. A three hinged stiffening girder of suspension bridges of span 100 m is subjected to two point loads of 200 kN and 300 kN at the distance of 25 m and 50 m from the left end. Find the shear force and bending moment for the girder at a distance 30 m from left end. If supporting cable has the central dip of 10 m, find the maximum tension in the cable.

Ans. Refer Q. 3.6, Page 3–11B, Unit-3.

4. Analyze the beam given in Fig. 5 by slope deflection method and draw its bending moment diagram and shear force diagram.

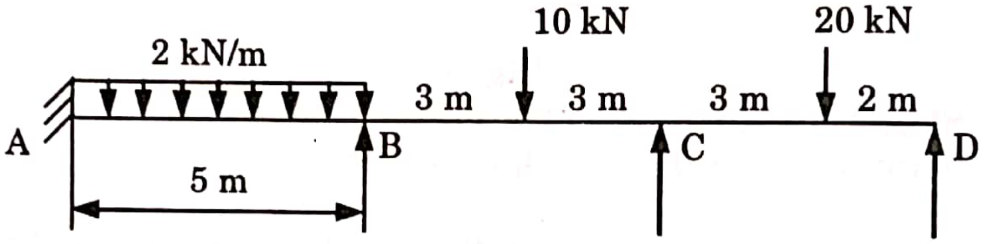


Fig. 5.

Ans. Refer Q. 1.7, Page 1–11B, Unit-1.

5. Develop the flexibility matrix for the cantilever with co-ordinate as shown in Fig. 6, take uniform flexural rigidity.

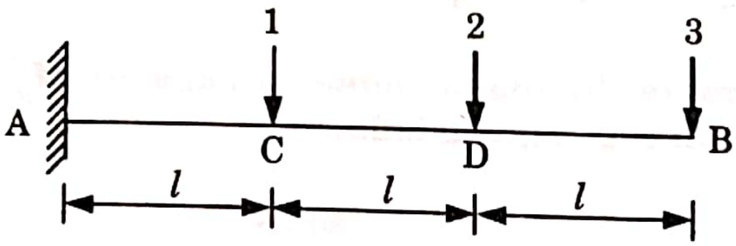


Fig. 6.

Ans. Refer Q. 4.2, Page 4–3B, Unit-4.

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B.Tech.

(SEM. V) ODD SEMESTER THEORY EXAMINATION, 2017-18
STRUCTURAL ANALYSIS-II

Time : 3 Hours **Max. Marks : 100**

Note : Attempt **all** sections. If any missing data required, then choose suitably.

Section – A

1. Attempt **all** questions in brief. **(2 × 10 = 20)**

a. **What do you mean by carry over moment ?**

Ans. Refer Q. 1.10, 2 Marks Questions, Page SQ–2B, Unit-1.

b. **State Castigliano's second theorem.**

Ans. Refer Q. 2.4, 2 Marks Questions, Page SQ–5B, Unit-2.

c. **Two hinged arch is a indeterminate structure, why ?**

Ans. Refer Q. 2.16, 2 Marks Questions, Page SQ–8B, Unit-2.

d. **How horizontal thrust can be obtained by using Castigliano's theorem ?**

Ans. Refer Q. 2.5, 2 Marks Questions, Page SQ–6B, Unit-2.

e. **What is the range of central dip ?**

Ans. The central dip of a cable ranges from 1/10 to 1/15 of the span.

f. **What is the effect of temperature change in cable ?**

Ans. Following are the effects of temperature change in cable :

i. Temperature increases causes loss of tension in the cables. The latter is susceptible of causing loss of stability.

ii. Low temperatures and the loss of ductility of certain materials could result in catastrophic failure.

iii. Due to rise in temperature, the length and dip of cable increases and reduces the horizontal thrust.

g. **Define stiffness coefficients.**

Ans. Refer Q. 4.5, 2 Marks Questions, Page SQ–13B, Unit-4.

h. **What are the objectives of analysis of structure ?**

Ans. Refer Q. 1.18, 2 Marks Questions, Page SQ–4B, Unit-1.

i. **Differentiate between plastic modulus and section modulus.**

Ans. Refer Q. 5.6, 2 Marks Questions, Page SQ–15B, Unit-5.

j. **What do you mean by plastic hinge ?**

Ans. Refer Q. 5.3, 2 Marks Questions, Page SQ–15B, Unit-5.

Section-B

2. Attempt any **three** of the following : **(3 × 10 = 30)**

a. **Find the moments at the supports for the continuous beam shown in Fig. 1. Draw also BM diagram for the beam. Use slope deflection method.**

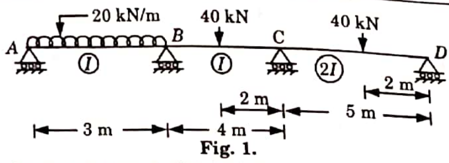


Fig. 1.

Ans. Refer Q. 1.8, Page 1–14B, Unit-1.

b. Using Muller Breslau Principle, compute the influence line ordinates at 2 m intervals for moment at mid span of *BC* of the continuous beam *ABC* shown in Fig. 2.

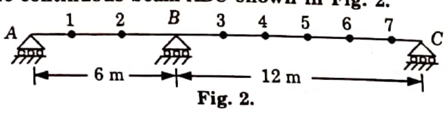


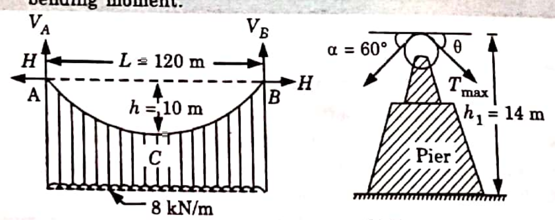
Fig. 2.

Ans. Refer Q. 2.4, Page 2–10B, Unit-1.

c. A cable of span 120 m and dip 10 m carries a load of 8 kN/m on the horizontal span. Find the maximum tension in the cable and the inclination of the cable at the support. Find also the forces transmitting to the supporting pier, if the cable passes over smooth pulley on the top of the pier. The anchor cable is at 30° to the vertical. Determine the maximum bending moment for the pier, if the height of pier is 14 m.

Ans.

Given : Span, L = 120 m, Dip, h = 10 m,
Intensity of load, w = 8 kN/m
Height of pier, h_1 = 14 m, Angle of cable, α = 30° (from vertical)
To Find : Maximum tension, Transmitted force and Maximum bending moment.



(*a*) Given cable. (*b*) Forces on pier

Fig. 3.

1. Reaction at the Supports :

i. Due to symmetry vertical reactions,

$$V_A = V_B = \frac{wL}{2} = \frac{8 \times 120}{2} = 480 \text{ kN}$$

ii. To obtain '*H*' taking moment about central point '*C*' (considering left part of *C*), $\Sigma M_C = 0$

$V_A \times 60 - H \times 10 - 8 \times 60 \times 60/2 = 0$

$480 \times 60 - 10\,H - 4 \times 60 \times 60 = 0$

$10\,H = 14400$

Horizontal reaction, $H = \frac{14400}{10} = 1440$ kN

2. Maximum Tension in the Cable :

i. Maximum tension will be occurs either supports A or B

$$T_{max} = \sqrt{(V_A)^2 + (H)^2} = \sqrt{(480)^2 + (1440)^2} = 1517.9 \text{ kN}$$

ii. $$\cos\theta = \frac{H}{T_{max}} = \frac{1440}{1517.9} = 0.9487$$

$$\theta = \cos^{-1}(0.9487) = 18.435°$$

3. Forces Transmitting to the Supporting Pier :

i. Referring to Fig. 3(*b*), horizontal force transmitted to pier

$= T_{max} (\cos\theta - \cos\alpha)$

$= 1517.9 (\cos 18.435° - \cos 60°) = 681.06$ kN

ii. Vertical force on the pier $= T(\sin\theta + \sin\alpha)$

$= 1517.9 (\sin 18.435° + \sin 60°) = 1794.54$ kN

4. Maximum Bending Moment in the Pier :

Maximum bending moment in the pier = Horizontal force on pier × Height of pier = 681.06 × 14 = 9534.84 kN-m

d. Analyse the continuous beam as shown in Fig. 4. If the downward settlement of supports B and C are 10 mm and 5 mm respectively. Take *EI* = 184 × 10^11 N-mm². Use flexibility matrix method.

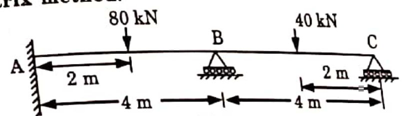


Fig. 4.

Ans. Refer Q. 4.7, Page 4–17B, Unit-4.

e. A two span continuous beam has span lengths *AB* = 6 m and *BC* = 6 m and carries a UDL of 30 kN/m on entire length of the beam. A and C are simply supports. If the load factor is 1.80 and the shape factor is 1.15 for the I-section. Find the section modulus needed. Assume yield stress for the material is 250 N/mm².

Ans. Refer Q. 5.14, Page 5–19B, Unit-5.

Section-C

3. Attempt any **one** part of the following : **(1 × 10 = 10)**

a. A continuous beam *ABCD* is simply supported at *A, B, C* and is fixed at *D*. The span *AB, BC* and *CD* are 3 m, 4 m and 2 m long. The beam carries a point load of 12 kN on *AB* at 2 m from A, a point load of 20 kN at the middle of *BC* and a point load of 6 kN at middle of *CD*. If $I_{ab} : I_{bc} : I_{cd}$ = 1:2:2. Find the supports moments using moment distribution method.

Ans. Refer Q. 1.16, Page 1–31B, Unit-1.

b. Analyse the following continuous beam shown in Fig. 5 using method of consistent deformation. Draw the bending moment diagram.

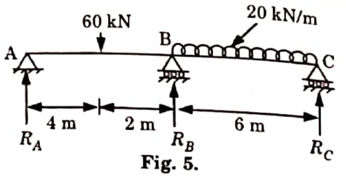


Fig. 5.

Ans.

1. **Degree of Static Indeterminacy :**
   i. Total number of reaction component, $R = 3 + 1 = 4$
   ii. Total number of static equilibrium equations, $E = 3$
   iii. Degree of static indeterminacy, $X = R - E = 4 - 3 = 1$
2. **Basic Determinate Structure :** To obtain basic determinate structure removing redundant force R_B. The resulting determinate structure is a simple supported beam ABC as shown in Fig. 6.

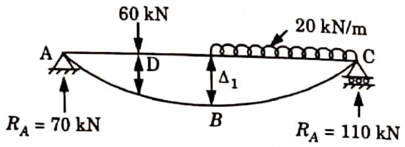


Fig. 6. Deflected shape of basic determinate beam due to external loads.

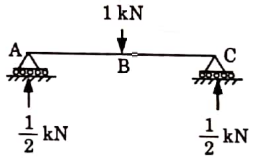


Fig. 7. Basic determinate beam subjected to unit load at B.

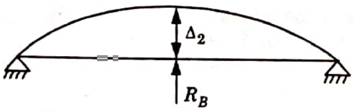


Fig. 8. Deflected shape of basic determinate beam due to redundant force R_B.

3. **Deflection under the Redundant Force :**
   i. To find the deflection at B in this continuous beam, the unit load method is used.
   ii. Reaction at supports in the determinate beam AC.
   Taking moment about 'C'

$$\Sigma M_C = 0$$
$$R_A \times 12 - 60 \times 8 - 20 \times 6 \times 3 = 0$$
$$R_A = 70 \text{ kN}$$

iii.
$$\Sigma V = 0$$
$$R_A - 60 - 20 \times 6 + R_C = 0$$
$$R_C = 60 + 20 \times 6 - 70 = 110 \text{ kN}$$

iv. Reactions at supports due to unit load at B, $R_A = R_C = \frac{1}{2}$ kN

v. Moments in various portions are shown in Table 1.

Table 1 : Moments in various portions of basic determinate beam.

Portion	AD	DB	DC
Origin	A	A	C
Limit	0 - 4	4 - 6	0 - 6
M	$70x$	$70x - 60(x - 4) = 10x + 240$	$110x - 10x^2$
m	$x/2$	$x/2$	$x/2$

vi. Deflection at B,

$$\Delta_1 = \int_0^4 70x \times \frac{x}{2} \times \frac{dx}{EI} + \int_4^6 10x \times \frac{x}{2} \times \frac{dx}{EI} + \int_4^6 240 \times \frac{x}{2} \times \frac{dx}{EI}$$
$$+ \int_0^6 110x \times \frac{x}{2} \times \frac{dx}{EI} - \int_0^6 10x^2 \times \frac{x}{2} \times \frac{dx}{EI}$$
$$= \frac{35}{3\,EI}[x^3]_0^4 + \frac{5}{3\,EI}[x^3]_4^6 + \frac{120}{2\,EI} + [x^2]_4^6 + \frac{55}{3\,EI}[x^3]_0^6 - \frac{5}{4\,EI}[x^4]_0^6$$
$$= \frac{746.67 + 1453.33 + 3960 - 1620}{EI}$$
$$= \frac{4540}{EI} (\downarrow)$$

vii. Upward deflection at B due to R_B (Fig. 8).

$$\Delta_2 = R_B\left[\frac{12^3}{48\,EI}\right](\uparrow)$$

4. **Consistency Requirements :**
   i. From the consistency condition,

$$\Delta_1 = \Delta_2$$
$$\frac{4540}{EI} = R_B \frac{12^3}{48\,EI}$$
$$R_B = 126.11 \text{ kN}$$

   ii. Therefore in the continuous beam ABC (Fig. 5).
   Taking the moment about 'C'

$$\Sigma M_C = 0$$
$$R_A \times 12 - 60 \times 8 + R_B \times 6 - 20 \times 6 \times 3 = 0$$
$$R_A \times 12 - 480 + 126.11 \times 6 - 360 = 0$$
$$R_A = 6.94 \text{ kN}$$

iii.
$$\Sigma V = 0$$

$$R_A + R_B + R_B = 60 + 20 \times 6 = 180 \text{ kN}$$
$$R_C = 180 - 6.94 - 126.11$$
$$R_C = 46.95 \text{ kN}$$

iv. Moment at '*B*' (Left part) :

$$R_A \times 6 - 60 \times 2 = 6.94 \times 6 - 60 \times 2 = -78.36 \text{ kN-m}$$

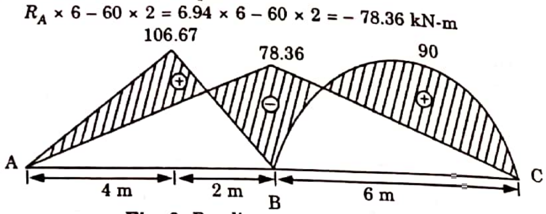


Fig. 9. Bending moment diagram.

4. Attempt any one part of the following : (1 × 10 = 10)

a. **A two hinged parabolic arch of span 30 m and rise 6 m carries two point loads, each 60 kN, acting at 22.5 m and 15 m from the right end respectively. Determine the horizontal thrust and maximum positive and negative moment in the arch.**

Ans. Refer Q. 2.9, Page 2–21B, Unit-2.

b. **A two hinged semicircular arch of radius '*R*' carried a load '*W*' at a section the radius vector corresponding to which makes an angle 'α' with the horizontal. Find the horizontal thrust at each support. Assume uniform flexural rigidity.**

Ans. Refer Q. 2.6, Page 2–15B, Unit-2.

5. Attempt any **one** part of the following : (1 × 10 = 10)

a. **A suspension cable of length 174.53 m is supported at the two ends at the same levels. The supports are 170 m apart. The cable is subjected to a UDL of 20 kN/m of horizontal length over its entire span. Determine the reactions developed at the supports and their inclination to the horizontal.**

Ans.

Given : Length of cable l, 174.53 m, Span of cable, L = 170 m, Intensity of UDL, w = 20 kN/m
To Find : Reaction of support and inclinations.

1. The cable is shown in Fig. 10. The length of the cable 'l' is given by,

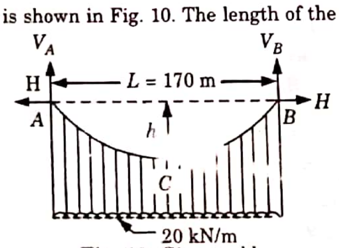


Fig. 10. Given cable.

$$l = L + \frac{8}{3} \times \frac{h^2}{L}$$

$$170 + \frac{8}{3} \times \frac{h^2}{170} = 174.53 \text{ m}$$

$$h = \sqrt{\frac{4.53 \times 170 \times 3}{8}} = 16.99 \approx 17 \text{ m}$$

2. Let 'H' be the horizontal force and V_A be the vertical reaction at 'A', then

$$V_A = \frac{wL}{2} = \frac{20 \times 170}{2} = 1700 \text{ kN}$$

3. **Maximum Tension in the Cable :** Maximum tension in the cable will be occurs either support *A* or *B* and is given by,

$$T_{max} = \sqrt{(V_A)^2 + (H)^2}$$

Horizontal reaction,

$$H = \frac{wL^2}{8h} = \frac{20 \times (170)^2}{8 \times 17} = 4250 \text{ kN}$$

$$= \sqrt{(1700)^2 + (4250)^2} = 4577.4 \text{ kN}$$

4. Its inclination to the horizontal is given by,

$$T_{max} \cos \theta = H$$

$$\therefore \quad \theta = \cos^{-1}\left[\frac{H}{T_{max}}\right] = \cos^{-1}\left(\frac{4250}{4577.4}\right)$$

$$\theta = 21°48' \text{ m}$$

b. **A three hinged stiffening girder of a suspension bridge of span 120 m is subjected to two point loads of 240 kN and 300 kN at distances of 25 m and 80 m respectively from the left end. Find the forces and bending moment in girder at a section 40 m from the left end. The central dip of the supporting cable is 12 m. Find also the maximum tension in the cable and draw the BMD for the girder.**

Ans. Refer Q. 3.7, Page 3–13B, Unit-3.

6. Attempt any **one** part of the following : (1 × 10 = 10)

a. **Develop the flexibility matrix for the cantilever with co-ordinate as shown in Fig. 11. Take uniform flexural rigidity.**

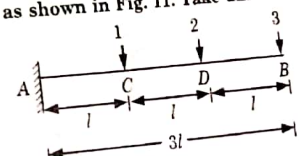


Fig. 11.

Ans. Refer Q. 4.2, Page 4–3B, Unit-4.

b. Analyse the continuous beam as shown in Fig. 12 by stiffness matrix method if the support *B* sink by 10 mm. Take EI = 6000 kN-m².

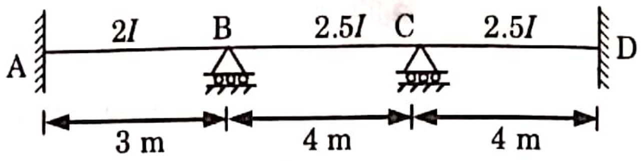


Fig. 12.

Ans. Refer Q. 4.12, Page 4–30B, Unit-4.

7. Attempt any **one** part of the following : **(1 × 10 = 10)**

a. Find the collapse load for the loaded portal frame shown in Fig. 13.

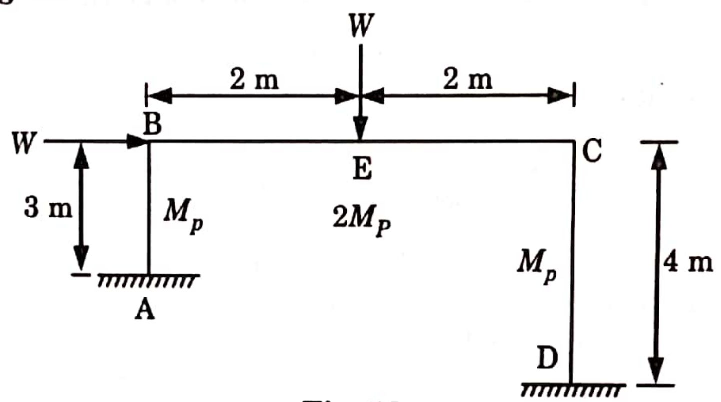


Fig. 13.

Ans. Refer Q. 5.18, Page 5–27B, Unit-5.

b. A continuous beam *ABC* is loaded as shown in Fig. 14, determine the required plastic moment. If the load factor is 3.2.

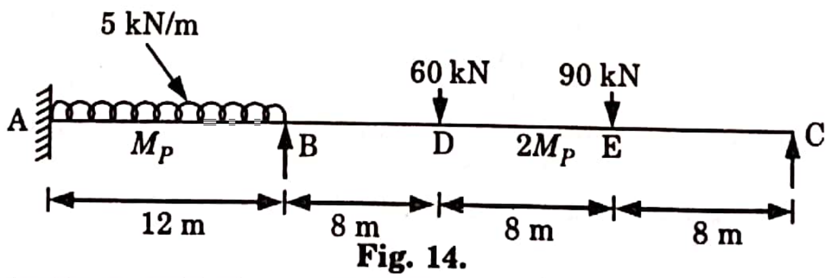


Fig. 14.

Ans. Refer Q. 5.15, Page 5–21B, Unit-5.

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B.Tech.

(SEM. V) ODD SEMESTER THEORY EXAMINATION, 2018-19
DESIGN OF STRUCTURE-I

Time : 3 Hours **Max. Marks : 70**

Note : Attempt **all** sections. Assume any missing data.

SECTION-A

1. Attempt **all** questions in brief. **(2 × 7 = 14)**

a. **What do you mean by degree of redundancy ?**

Ans. Refer Q. 4,1, 2 Marks Questions, Page SQ–12B, Unit-4.

b. **Write the assumptions are made while developing slope deflection method.**

Ans. Following are the assumptions made for developing the slope deflection method :

i. All the joints of the frame are rigid.

ii. Distortion, due to axial and shear stresses, being very small, are neglected.

c. **What is the effect of temperature change in the cable ?**

Ans. Refer Q. 2(f), Page SP–20B, Solved Paper 2017-18.

d. **Write different approaches to matrix method.**

Ans. There are two approaches to the solution of matrix method :

1. Direct approach.
2. Transformation matrix approach.

e. **What is restrained structure ?**

Ans. In structural engineering the term "restraint" is used to denote how much a structure is slopped from buckling sideways.

f. **Differentiate between plastic modulus and section modulus.**

Ans. Refer Q. 5.6, Page SQ–15B, Unit-5.

g. **Write the limitations of load factor concept.**

Ans. Limitations of Load Factor Concept :

i. This method leads to excessive deformation and cracking.

ii. No factors of safety are used for material stresses.

SECTION-B

2. Attempt any **three** of the following : **(7 × 3 = 21)**

a. **A continuous beam ABCD is load as shown in Fig. 1. During loading support B sinks by 1 cm. Determine the support moments. Take $I = 1600$ cm^4; $E = 2 \times 10^5$ N/mm^2. Use moment distribution method.**

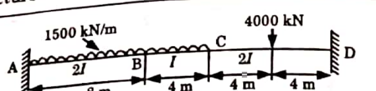


Fig. 1.

Ans.

1. **Fixed End Moments :**

$EI = 1600 \times 10^4 \times 2 \times 10^5 = 32 \times 10^{11}$ N-mm² $= 32 \times 10^2$ kN-m²

$\Delta = 1$ cm $= 0.01$ m

$$M^F_{AB} = -\frac{wL^2}{12} - \frac{6E(2I)\Delta}{L^2} = -\frac{1500 \times 8^2}{12} - \frac{6 \times (2 \times 32 \times 10^2) \times 0.01}{8^2}$$
$$= -8006 \text{ kN-m}$$

$$M^F_{BA} = \frac{wL^2}{12} - \frac{6E(2I)\Delta}{L^2} = \frac{1500 \times 8^2}{12} - \frac{6 \times (2 \times 32 \times 10^2) \times 0.01}{8^2}$$
$$= 7994 \text{ kN-m}$$

$$M^F_{BC} = -\frac{wL^2}{12} + \frac{6EI\Delta}{L^2} = -\frac{1500 \times 4^2}{12} + \frac{6 \times 32 \times 10^2 \times 0.01}{4^2}$$
$$= -1998 \text{ kN-m}$$

$$M^F_{CB} = \frac{wL^2}{12} + \frac{6EI\Delta}{L^2} = \frac{1500 \times 4^2}{12} + \frac{6 \times 32 \times 10^2 \times 0.01}{4^2} = 2012 \text{ kN-m}$$

$$M_{CD} = -\frac{Wab}{L} = -\frac{4000 \times 4 \times 4}{8} = -8000 \text{ kN-m}$$

$$M_{DC} = +\frac{Wab}{L} = +\frac{4000 \times 4 \times 4}{8} = +8000 \text{ kN-m}$$

2. **Distribution Factors :**

Joint	Member	Relative Stiffness	Total Relative Stiffness	Distribution Factor
B	BA	$4E(2I)/8 = EI$	$2EI$	0.5
	BC	$4EI/4 = EI$		0.5
C	CB	$4EI/4 = EI$	$2EI$	0.5
	CD	$4E(2I)/8 = EI$		0.5

3. **Moment Distribution :**

	A	B	B	C	C	D
		0.5	0.5	0.5	0.5	
FEM	– 8806	7994	– 1988	2012	– 8000	8000
Balancing		– 3003	– 3003	2994	2994	
Final moment	– 8006	4991	– 4991	5006	– 5006	8000

b. A two hinged semicircular arch of radius *R* carries a concentrated load *W* at the crown. Find the vertical deflection of the crown. Assume uniform flexural rigidity.

Ans.

Given : Radius of arch = *R*, Load = W
To Find : Vertical deflection at the crown.

1. Consider the arch as shown in Fig. 2.

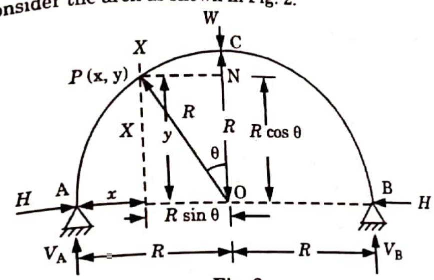


Fig. 2.

2. Due to symmetry,
$$V_A = V_B = \frac{W}{2}$$
3. From ΔOPN, $y = R \cos \theta$
4. $x = R - R \sin \theta = R(1 - \sin \theta)$
5. We know that, horizontal thrust at the crown, $H = \frac{W}{\pi}$
6. Bending moment at any section *x–x*,
$$M = \frac{W}{2}x - \frac{W}{\pi}y = \frac{W}{2}R(1 - \sin\theta) - \frac{W}{\pi}R\cos\theta$$
7. Strain energy stored,
$$W_i = \int \frac{M^2}{2EI} ds = 2\int_0^{\pi/2} \frac{\left[\frac{W}{2}R(1-\sin\theta) - \frac{W}{\pi}R\cos\theta\right]^2}{2EI} Rd\theta$$
$$= \frac{1}{EI}\int_0^{\pi/2}\left[\frac{R}{2}(1-\sin\theta) - \frac{R\cos\theta}{\pi}\right]^2 W^2 Rd\theta$$
8. Vertical deflection at the crown,
$$\delta = \frac{dW_i}{dW} = \frac{1}{EI}\int_0^{\pi/2} R^2\left[\frac{(1-\sin\theta)}{2} - \frac{\cos\theta}{\pi}\right]^2 2W\, Rd\theta$$
$$\delta = \frac{2WR^3}{EI} \times 0.048 = \frac{0.096\, WR^3}{EI} \text{ unit}$$

c. The three hinged girder of a suspension bridge have a span of 200 m, the dip of the supporting cable being 16 m. If the girder is subjected to two point loads 450 kN and 280 kN at distances of 40 m and 150 m from the left end. Find the SF and BM for the girder at 75 m from the left end. Find also the maximum tension in the cable. Draw the bending moment diagram for the girder.

Ans.

Given : Span of bridge, L = 200 m, Points loads = 450 kN and 280 kN, Central dip, h = 16 m, Distance of section from end A = 75 m
To Find : Maximum tension, shear force and bending moment at section, Draw BMB.

1. To calculate vertical reactions, $\Sigma V = 0$
$$V_A + V_B = 450 + 280 = 730 \text{ kN}$$
2. Taking moment about support B, we get
$$V_A \times 200 = 450 \times 160 + 280 \times 50$$
$$V_A = 430 \text{ kN}$$
$$V_B = 730 - 430 = 300 \text{ kN}$$

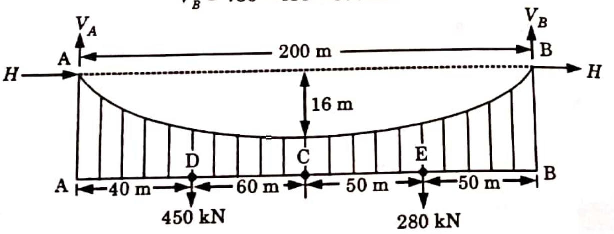


Fig. 3.

3. Now taking moment about central hinged C, $\Sigma M_C = 0$
$$V_A \times 100 = 450 \times 60 + H \times 16$$
$$430 \times 100 = 450 \times 60 + H \times 16$$
$$H = 1000 \text{ kN}$$
4. Now,
$$H = \frac{wL^2}{8h}$$
$$1000 = \frac{w \times 200^2}{8 \times 16}$$
UDL on cable, w = 3.2 kN/m
5. Vertical reaction for the cable, $V = \frac{wL}{2} = \frac{3.2 \times 200}{2}$ = 320 kN
6. Maximum tension in the cable,
$$T_{max} = \sqrt{V^2 + H^2} = \sqrt{320^2 + 1000^2} = 1049.95 \text{ kN}$$

7. The bending moment at the section at a distance 75 m from left hinged
$$= \text{Beam BM} - H_{moment}$$
8. Beam moment at distance 30 m from left hinged
$$= V_A \times 75 - 450 \times 35 = 430 \times 75 - 450 \times 35 = 16500 \text{ kN-m}$$
9. $H_{moment} = Hy$
$$y = \frac{4hx\,(L - x)}{L^2} = \frac{4 \times 16 \times 75\,(200 - 75)}{200^2} = 15 \text{ m}$$
$$H_{moment} = 1000 \times 15 = 15000 \text{ kN-m}$$
10. Actual bending moment at section = Beam bending moment – Hy
$$= 16500 - 15000 = 1500 \text{ kN-m}$$
11. Shear force at the section at a distance 75 m from left hinged,
$$\text{SF} = \text{Beam shear} - H \tan\theta$$
12. Beam shear at section (x = 75 m) = V_A – 450 = 430 – 450 = – 20 kN
13. $$\tan\theta = \frac{dy}{dx} = \frac{4h}{L^2}(L - 2x)$$
$$\tan\theta = 4 \times 16 \times (200 - 2 \times 75)/\, 200^2 = 0.08$$
14. Actual shear force at section = Beam shear – $H \tan\theta$
$$= -20 - 1000 \times 0.08$$
Shear force = – 100 kN
15. Actual BM on girder :
i. At Point D :
$$y_D = \frac{4hx}{L^2}(L - x) = \frac{4 \times 16 \times 40}{200^2}(200 - 40) = 10.24 \text{ m}$$
$$BM_D = 430 \times 40 - 1000 \times 10.24 = 6960 \text{ kN-m}$$
ii. At Point C :
$$y_C = \frac{4hx}{L^2}(L - x) = \frac{4 \times 16 \times 100}{200^2}(200 - 100) = 16 \text{ m}$$
$$BM_C = 430 \times 100 - 450 \times 60 - 1000 \times 16 = 0$$
iii. At Point E :
$$y_E = \frac{4 \times 16 \times 150}{200^2}(200 - 150) = 12 \text{ m}$$
$$BM_E = -300 \times 50 + 1000 \times 12 = -3000 \text{ kN-m}$$

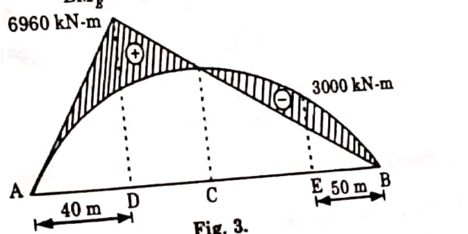


Fig. 3.

d. What is stiffness matrix ? Give the step of flexibility matrix method for analysis of indeterminate beam.

Ans. **Stiffness Matrix :** Refer Q. 4.8, Page 4–21B, Unit-4.
Steps : Refer Q. 4.8, Page 4–21B, Unit-4.

e. **Find the shape factor of the I-section shown in Fig. 4.**

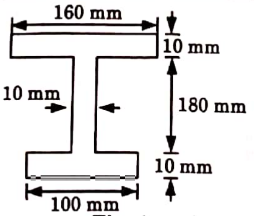


Fig. 4.

Ans.

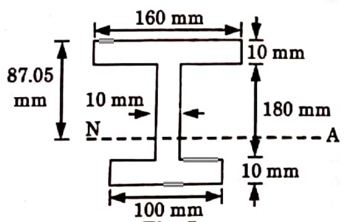


Fig. 5.

1. Elastic neutral axis from top fibre,

$$y_t = \frac{160 \times 10 \times 5 + 180 \times 10 \times (90 + 10) + 100 \times 10 \times (195)}{160 \times 10 + 180 \times 10 + 100 \times 10}$$

$$= 87.05 \text{ mm}$$

2. Moment of inertia about elastic neutral axis,

$$I = \frac{1}{12} \times 160 \times 10^3 + 160 \times 10(87.05 - 5)^2$$

$$+ \frac{1}{12} \times 10 \times 180^3 + 10 \times 180(100 - 87.05)^2$$

$$+ \frac{1}{12} \times 100 \times 10^3 + 100 \times 10(195 - 87.05)^2$$

$$= 10784857.33 + 5161864.5 + 11661535.83$$

$$I = 27608257.66 \text{ mm}^4$$

3. Distance of extreme fibre,

$$y_{max} = 200 - 87.05 = 112.95 \text{ mm}$$

4. Elastic moment, $M_y = f_y\left(\frac{I}{y_{max}}\right) = f_y\left(\frac{27608257.66}{112.95}\right) = 244429.02 f_y$

5. **Plastic Moment (M_p) :**

i. Let plastic neutral axis be at a distance y_p from bottom fibre. Assuming it fall in web.

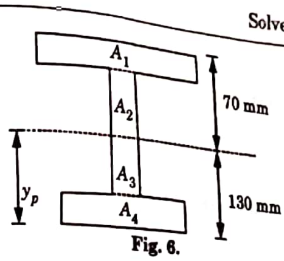


Fig. 6.

$$100 \times 10 + 10(y_p - 10) = \frac{(160 + 180 + 100) \times 10}{2}$$

$$10(y_p - 10) = 2200 - 1000$$

$$y_p = 130 \text{ mm}$$

The assumption that y_p is in web is correct.

ii. Dividing the total area into four rectangular, two in compression zone and two in tension zone.

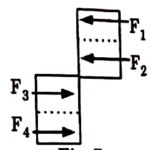


Fig. 7.

iii. Plastic moment is given by, $M_p = f_y \Sigma Ay$

$$= \left[160 \times 10 \times (70 - 5) + 10 \times (70 - 10) \times \frac{(70 - 10)}{2} + 10 \times (130 - 10) \times \frac{(130 - 10)}{2} + 100 \times 10 \times (130 - 5)\right]$$

$$M_p = 319000 f_y \text{ N-mm}$$

6. Shape factor, $S = \frac{M_p}{M_y} = \frac{319000 f_y}{244429.02 f_y} = 1.305$

SECTION–C

3. Attempt any one part of the following : **(7 × 1 = 7)**

a. **Analyse the frame shown in Fig. 8 by slope deflection method.**

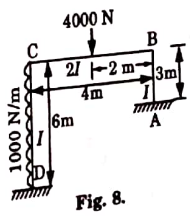


Fig. 8.

Ans.

1. **Fixed End Moments :**

$$M_{AB}^F = M_{BA}^F = 0$$

$$M_{BC}^F = \frac{WL}{8} = \frac{4000 \times 4}{8} = 2 \text{ kN-m}$$

$$M_{CB}^F = -\frac{WL}{8} = \frac{4000 \times 4}{8} = -2 \text{ kN-m}$$

$$M_{CD}^F = \frac{wl^2}{12} = \frac{1000 \times 6^2}{12} = 3000 \text{ N-m} = 3 \text{ kN-m}$$

$$M_{DC}^F = -\frac{wl^2}{12} = -\frac{1000 \times 6^2}{12} = -3000 \text{ N-m} = -3 \text{ kN-m}$$

2. **Slope Deflection Equations :**

i. **For span *AB* :**

$$M_{AB} = M_{AB}^F + \frac{2EI}{L}(2\theta_A + \theta_B) = 0 + \frac{2EI}{3}\theta_B \qquad (\because \theta_A = 0)$$

$$M_{BA} = 0 + \frac{2EI}{L}(\theta_A + 2\theta_B) = \frac{2EI}{3} \times 2\theta_B = \frac{4}{3} EI\,\theta_B$$

ii. **For Span *BC* :**

$$M_{BC} = M_{BC}^F + \frac{2EI}{L}(2\theta_B + \theta_C) = 2 + \frac{2E \times (2I)}{4}(2\theta_B + \theta_C)$$

$$= 2 + EI\,(2\theta_B + \theta_C)$$

$$M_{CB} = M_{CB}^F + \frac{2EI}{L}(2\theta_C + \theta_B) \qquad = -2 + \frac{2E\,(2I)}{4}(2\theta_C + \theta_B)$$

$$= -2 + EI\,(2\theta_C + \theta_B)$$

iii. **For Span *CD* :**

$$M_{CD} = M_{CB}^F + \frac{2EI}{L}(2\theta_C + \theta_D) \qquad (\because \theta_D = 0)$$

$$M_{CD} = +\ 3 + \frac{2EI}{6}(2\theta_C + \theta_D) = 3 + \frac{2}{3}EI\,\theta_C$$

$$M_{DC} = M_{DC}^F + \frac{2EI}{L}(2\theta_D + \theta_C) = -\ 3 + \frac{2EI}{6}(0 + \theta_C) = -3 + \frac{EI}{3}\theta_C$$

3. **Equilibrium condition at *B* :**

$$M_{BA} + M_{BC} = 0$$

$$\frac{4}{3}EI\,\theta_B + 2 + 2\,EI\theta_B + EI\,\theta_C = 0$$

$$\frac{10}{3}EI\,\theta_B + EI\theta_C = -2 \qquad ...(1)$$

4. **Equilibrium Condition at *C* :**

i. $$M_{CB} + M_{CD} = 0$$

$$-2 + 2\,EI\,\theta_C + EI\,\theta_B + 3 + \frac{2}{3}EI\,\theta_C = 0$$

$$EI\,\theta_B + \frac{8}{3}EI\,\theta_C = -1 \qquad ...(2)$$

ii. From eq. (1) eq. (2), we get

$$EI\,\theta_B = -0.55, \quad EI\,\theta_C = -0.17$$

5. **End Moments Calculations :**

$$M_{AB} = \frac{2}{3}(-0.55) = -0.37 \text{ kN-m}$$

$$M_{BA} = \frac{4}{3}(-0.55) = -0.73 \text{ kN-m}$$

$$M_{BC} = 2 + 2(-0.55) + (-0.17) = 0.73 \text{ kN-m}$$

$$M_{CB} = -2 + 2(-0.17) - 0.55 = -2.89 \text{ kN-m}$$

$$M_{CD} = 3 + \frac{2}{3}(-0.17) = 2.89 \text{ kN-m}$$

$$M_{DC} = -\ 3 + \frac{(-0.17)}{3} = -3.056 \text{ kN-m}$$

6. **Bending Moment Diagram :**

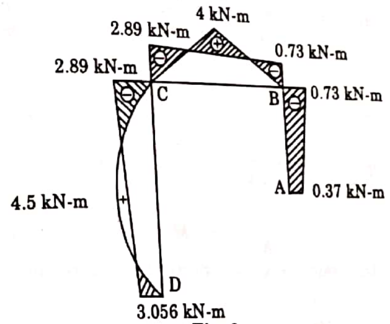


Fig. 9.

b. **Analyse the fixed beam shown in Fig. 10, using strain energy method. And draw the BMD.**

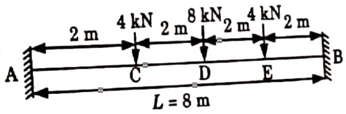


Fig. 10.

Ans.

1. **Reactions at Supports :** Due to symmetrically loaded beam, the reaction and moments at supports *A* and *B* are equal *i.e.*,

$$V_A = R_B$$

and $$M_A = M_B$$

Let $M_A = M_B = M$ as shown in Fig. 11.

Total load on beam = 4 + 8 + 4 = 16 kN.

$$\therefore \quad V_A = V_B = \frac{\text{Total load}}{2} = \frac{16}{2} = 8 \text{ kN}$$

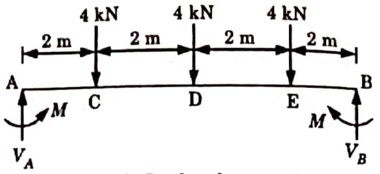


Fig. 11. Beam with fixed and moment at *A* and *B*.

2. **Fixing Moments at Supports :**

i. Due to symmetry strain energy in portion *AD* equals to strain energy in portion *DB*. Hence, total strain energy stored in the beam *AB*

$$= 2 \times \text{Strain energy in portion } AD$$

ii. Considering portion *AC* and taking origin at *A*, then

$$M_X = V_A x - M = 8x - M$$

iii. Considering portion *CD* and taking *C* as a origin, then

$$M_X = V_A(x + 2) - M - 4x = 8(x + 2) - M - 4x$$
$$= 8x + 16 - M - 4x = 4x + 16 - M$$

iv. $\therefore$ Strain energy stored in beam *AB*, $W_i = 2\int_0^l \frac{M_X^2\, ds}{2\,EI}$

$$W_i = 2\left[\int_0^2 (8x - M)^2 \frac{dx}{2\,EI} + 2\int_0^2 (4x + 16 - M)^2 \frac{dx}{2\,EI}\right]$$

$$= \frac{1}{EI}\int_0^2 (8x - M)^2\, dx + \frac{1}{EI}\int_0^2 (4x + 16 - M)^2\, dx$$

v. For total strain energy of deflected beam to be minimum,

$$\frac{\delta W_i}{\delta M} = 0$$

i.e., $\frac{1}{EI}\int_0^2 2(8x - M)(-1)dx + \frac{1}{EI}\int_0^2 2(4x + 16 - M)(-1)\, dx = 0$

$$-\frac{2}{EI} \times 8\left[\frac{x^2}{2}\right]_0^2 + \frac{2M}{EI}[x]_0^2 + \frac{2}{EI}\left[-4\left[\frac{x^2}{2}\right]_0^2 - 16[x]_0^2 + M[x]_0^2\right] = 0$$

$$-\frac{8}{EI} \times 4 + \frac{4M}{EI} - \frac{16}{EI} - \frac{32}{EI} + \frac{2M}{EI} = 0$$

$$6M = 80$$

$$M = \frac{80}{6} = 13.33 \text{ kN-m}$$

Hence BM at *A* and *B* = 13.33 kN-m

3. **Free BM Calculation :**

i. Free BM at *A* and *B* = 0

ii. Free BM at *C* = 8 × 2 = 16 kN-m

iii. Free BM at *D* = 8 × 4 – 4 × 2 = 24 kN-m

iv. Free BM at *E* = 8 × 2 = 16 kN-m

4. **BMD :** The BM diagram is shown in Fig. 12.

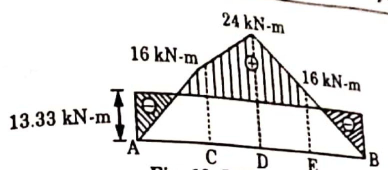


Fig. 12. BMD.

4. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. **A two hinged parabolic arch has span of 20 m and a rise 5 m carries a UDL of 20 kN/m for a distance of 5 m from the left end. Determine.**

i. **The horizontal thrust at each support.**

ii. **Bending moment, normal thrust and radial shear at a section of the arch 5 m from the left end.**

Ans.

Given : Span of arch, L = 20 m, Rise, h = 5 m, Intensity of UDL, w = 20 kN/m, Length of UDL = 5 m.

To Find : Horizontal thrust, Bending moment, Normal thrust and radial shear at section.

1. Vertical reactions, $\Sigma V = 0$

$$V_A + V_B = 20 \times 5 = 100 \text{ kN} \quad ...(1)$$

2. Taking moment about support *B*, $\Sigma M_B = 0$

$$V_A \times 20 - 20 \times 5 \times (5/2 + 15) = 0$$
$$V_A = 87.5 \text{ kN}$$

3. From eq. (1), $V_A + V_B = 100$

$$V_B = 100 - 87.5 = 12.5 \text{ kN}$$

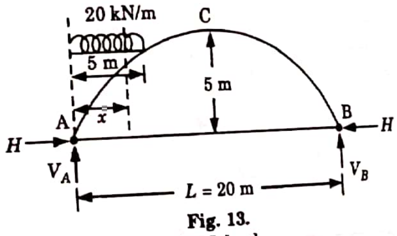


Fig. 13.

4. Horizontal reaction for UDL load,

$$H = \frac{1}{16}\frac{w}{hL^3}a^2(5L^3 - 5La^2 + 2a^3)$$

$$= \frac{1}{16} \times \frac{20}{5 \times 20^3} \times (5)^2\ [5 \times 20^3 - 5 \times 20 \times 5^2 + 2(5)^3] = 29.5 \text{ kN}$$

5. Bending moment at section at a distance 5 m from left end, ...(2)

$$M_x = \text{Beam moment} - \text{H moment}$$
$$M_x = V_A \times 5 - 20 \times 5 \times (5/2) - Hy$$

$$y = \frac{4 \times 5 \times 5}{20^2}(20 - 5) = 3.75 \text{ m}$$

$$M_x = 87.5 \times 5 - 20 \times (5^2/2) - 29.5 \times 3.75 = 76.875 \text{ kN-m}$$

6. Normal Thrust and Radial Shear :

i. $\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left[\frac{4hx(L-x)}{L^2}\right]$

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{L^2}(L - 2x) = \frac{4 \times 5}{20^2}(20 - 2 \times 5) = 0.5$$

$$\theta = 26° 34'$$

ii. Normal thrust, $N = V \sin\theta + H\cos\theta$ ...(3)

$$N = (87.5 - 20 \times 5) \sin 26° 34' + 29.5 \times \cos 26° 34' = 20.8 \text{ kN-m}$$

iii. Radial shear, $S = V\cos\theta - H\sin\theta$

$$= (87.5 - 20 \times 5) \cos 26° 34' - 29.5 \sin 26° 34' = -24.4 \text{ kN}$$

b. Using Muller Breslau principle, compute the influence line obtained at 2 m intervals for reaction at C of the continuous beam ABC shown in Fig. 14 below :

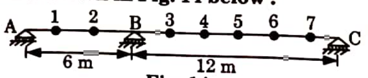


Fig. 14.

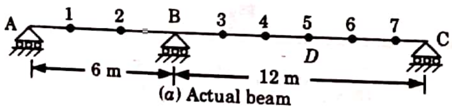
Ans.

1. Restrain in the direction of R_C is removed and unit load in the direction of R_C is applied as shown in Fig. 15.
2. According to Muller-Breslau principle, the defected shape of this beam is the influence line diagram for R_C.
3. The bending moment diagram for the released beam is as shown in Fig. 15(*c*).
4. To find the deflections in this released structure consider its conjugate beam with (M/EI) diagram as load shown in Fig. 15(*d*).
5. Summation of vertically forces, $\Sigma f_y = 0$

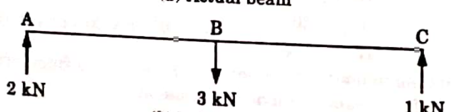
$$V_A + V_B = -1 \text{ kN}$$

6. Bending moment about A, $\Sigma M_A = 0$

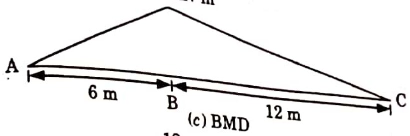
$$1 \times 18 + V_B 6 = 0$$
$$V_B = -3 \text{ kN}$$
$$V_A = 3 - 1 = 2 \text{ kN}$$



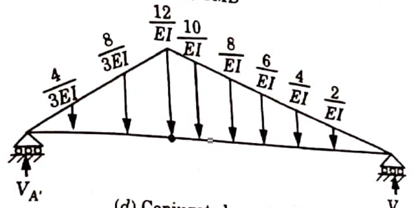
(*a*) Actual beam



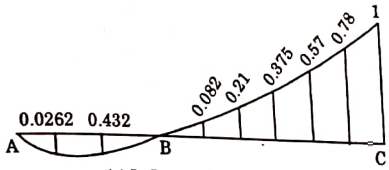
(*b*) Released beam



(c) BMD



(*d*) Conjugate beam with load



(*e*) Influence line diagram for V_C

Fig. 15.

7. In conjugate beam, $\Sigma M_{B'} = 0$, we get

$$V'_A \times 6 = (1/2) \times 6 \times 12 \times 6/(EI \times 3) = 12/EI$$

8. $\Sigma V = 0$

$$V'_A + V'_C = (1/2) \times 18 \times 12/EI$$
$$(12/EI) + V'_C = 108/EI$$
$$V'_C = 96/EI$$

9. Moment at point C in conjugate beam, $\Sigma M_C = 0$, we get

$$M'_C + (12/EI) \times 18 - (1/2) \times 18 \times (12/3EI) \times (18 + 12) = 0$$
$$M'_C = 864/EI$$

10. Taking moment causing tension at the bottom (sagging) as positive and the one causing tension at the top (hogging) as negative, we get

11. $M_C = -864/EI$

$$M_7 = -864/EI + (96/EI) \times 2 - (1/2) \times 2 \times (4/3EI) = -673.33/EI$$
$$M_6 = -864/EI + (96/EI) \times 4 - (1/2) \times 4 \times (16/3EI) = -490.67/EI$$
$$M_5 = -864/EI + (96/EI) \times 6 - (1/2) \times 6 \times (36/3EI) = -324/EI$$
$$M_4 = -864/EI + (96/EI) \times 8 - (1/2) \times 8 \times (64/3EI) = -181.33/EI$$
$$M_3 = -864/EI + (96/EI) \times 10 - (1/2) \times 10 \times (100/3EI) = -70.67/EI$$
$$M_B = -864/EI + (96/EI) \times 12 - (1/2) \times 12 \times (144/3EI) = 0$$

12. For portion AB, moments are calculated from the left side end.
$M_A = 0$
$M_1 = (12/EI) \times 2 - (1/2) \times 2 \times (4/3EI) = 22.67/EI$
$M_2 = (12/EI) \times 4 - (1/2) \times 4 \times (16/3EI) = 37.33/EI$
13. The above moments in the conjugate beam represent deflections in the released beam; and the deflections in the released beam represent the influence line diagram in the given beam.
14. Now, influence line diagram (ILD) ordinate at C
$= (864/EI)/(864/EI) = 1$
15. Hence, dividing moment values in conjugate beam by $864/EI$, we get ILD for V_C in the given beam.
16. ILD for V_C, thus obtained, is shown in Fig. 15(e).

5. Attempt any one part of the following : **(7 × 1 = 7)**

a. A suspension bridge has a cable of span 100 m and dip of 10 m. The cable is stiffened by a three hinged stiffening girder. Sketch the influence line diagram for bending moment at quarter span of girder. Determine the maximum moment at this section when a UDL longer than the span of intensity 10 kN/m traverses the span.

Ans.

Given : Span of bridge = 100 m, Dip = 10 m, Intensity of UDL, w = 10 kN/m
To Find : Draw ILD for Bending moment and maximum BM.

1. Fig. 16 shows influence line diagram for bending moment at quarter span D.

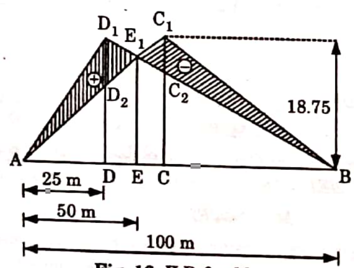


Fig. 16. ILD for M_D.

2. Maximum ordinate in this diagram is given by

$$DD_1 \text{ or } CC_1 = \frac{x(L-x)}{L} = \frac{25(100-25)}{100} = 18.75$$

3. Maximum moment occurs when the uniformly distributed load occupies portion AE. To locate E_1 we write expression for EE_1 from triangles AC_1B and AD_1B.

$$(AE/AC) \times CC_1 = EE_1 = (BE/BD) \times DD_1 \quad ...(1)$$

$(AE/50) \times 18.75 = (BE/75) \times 18.75$
$AE/BE = 50/75$
Adding 1 on both sides, we get
$1 + (AE/BE) = 1 + (50/75)$
$(BE + AE)/BE = (75 + 50)/75$
$AB/BE = 125/75$
$BE = (75/125) \times AB = (75/125) \times 100 = 40$ m

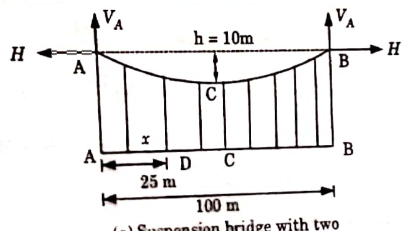
From eq. (1) ($\because AB = 100$ m)
$EE_1 = (40/50) \times 18.75 = 15$

4. Maximum positive moment,
M_{max} = Intensity of load × Area of ΔAD_1E_1
$= 10 \times [\Delta AD_1B - \Delta AE_1B]$
$= 10 \times [(1/2) \times 100 \times DD_1 - (1/2) \times 100 \times EE_1]$
$= 10 \times (1/2) \times 100(DD_1 - EE_1)$
$= 10 \times (1/2) \times 100(18.75 - 15) = 1875$ kN-m
5. Maximum negative moment $= 10[\Delta BC_1A - \Delta AE_1B]$
$= 10 \times [(1/2) \times 100 \times CC_1 - (1/2) \times 100 \times EE_1]$ ($\because DD_1 = CC_1$)
$= 10 \times [(1/2) \times 100 \times 18.75 - (1/2) \times 100 \times 15] = 1875$ kN-m

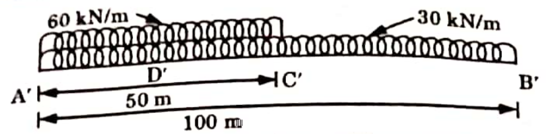
b. A suspension bridge of span 100 m and width 6 m is having two cables stiffened with hinged girders. The central dip of cable is 10 m. The dead load on the bridge is 10 kN/m² and live load is 20 kN/m² which covers the left half of the span. Determine the shear force and bending moment at 25 m from the left end. Find also the maximum tension in the cable.

Ans.

Given : Span, L = 100 m, Width of bridge = 6 m, Central dip = 10 m, section distance = 25 m, Intensity of dead load, w_d = 10 kN/m², Intensity of live load, w_L = 20 kN/m²
To Find : Shear force and Bending moment at section and Maximum tension in the cable.



(a) Suspension bridge with two hinged stiffening girder.



(b) Loading for beam shear and beam moment
Fig. 17.

1. **Total Lead on Girder :**
   i. Dead load per girder = 10 × (6/2) = 30 kN/m on full span.
   ii. Live load per girder = 20 × (6/2) = 60 kN/m on left half span.
   iii. Total load on girder,
   $$W = 30 \times 100 + 60 \times 50 = 3000 + 3000 = 6000 \text{ kN}$$
2. **Equivalent UDL from Cable :**
$$w_e = \frac{\text{Total load}}{\text{span}} = \frac{6000}{100} = 60 \text{ kN/m}$$
3. **Horizontal and Vertical Reactions at Supports :**
$$H = w_e L^2/8h = 60 \times 100^2/(8 \times 10) = 7500 \text{ kN}$$
$$V = w_e L/2 = 60 \times 100/(2) = 3000 \text{ kN}$$
4. **Maximum Tension in the Cable :**
$$T_{max} = \sqrt{V^2 + H^2} = \sqrt{3000^2 + 7500^2} = 8077.75 \text{ kN}$$
5. **Analysis of Girder :**
   i. Now consider the analysis of girder, which is subjected to a given load and to upward w_e at D' which is 25 m from left support
   ii. Reaction at A', $R_A' = \dfrac{\text{Moment about } B'}{100}$
   $$= \frac{30 \times 100 \times 50 + 60 \times 50 \times (100 - 25)}{100} = 3750 \text{ kN}$$
   iii. Beam shear at D' = 3750 – 30 × 25 – 60 × 25 = 1500 kN
   iv. Actual shear at $D' = 1500 - w_e\left(\dfrac{l}{2} - x\right) = 1500 - 60\left(\dfrac{100}{2} - 25\right) = 0$
   v. **Moment at Section :**
   $$M_D = \text{Beam moment + Moment due to } w_e$$
   $$= \text{Beam moment} - \frac{w_e \times x(L - x)}{2}$$
   $$= 3750 \times 25 - 30 \times 25 \times \frac{25}{2} - 60 \times 25 \times \frac{25}{2} - 60 \times 25 \frac{(100 - 25)}{2}$$
   $$= 9375 \text{ kN-m}$$

6. Attempt any one part of the following : **(7 × 1 = 7)**

a. **Analyse the continuous beam shown in Fig. 18 by flexibility matrix method. Take EI is constant.**

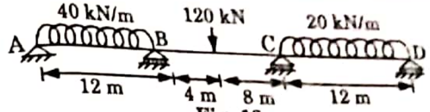
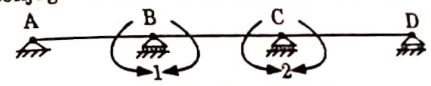
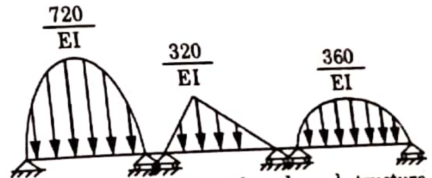


Fig. 18.

Ans.
1. The moments at B and C are taken as redundant forces.
2. Hence, released structure is obtained by introducing the hinges at B and C.
3. The coordinate directions chosen are as shown in Fig. 19(a).
4. **Displacements due to given loadings in the released structure :**
   i. The conjugate beam and the load on it are as shown in Fig. 19(b).

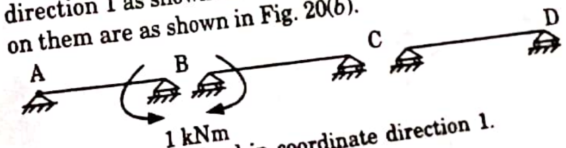


(a) Coordinate direction.

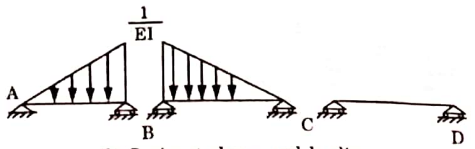


(b) Conjugate beam for released structure.
Fig. 19.

   ii. In conjugated beam, the maximum ordinate of load in
   $$AB = 40 \times 12^2/8EI = 720/EI$$
   iii. The maximum ordinate of load in
   $$BC = 120 \times 4 \times 8/12EI = 320EI$$
   iv. The maximum ordinate of load in
   $$CD = 20 \times 12^2/8EI = 360/EI$$
   v. Δ_{1L} = Displacement in coordinate direction 1 = Rotation of AB at B + Rotation of BC at B = Shear force in conjugate beam in AB at B + Shear force in conjugate beam in BC at B
   $$= \frac{1}{2} \times \frac{2}{3} \times \frac{720}{EI} \times 12 + \frac{1}{2} \times \frac{320}{EI} \times 12 \times \frac{(12 + 8)}{3} \times \frac{1}{12} = \frac{3946.67}{EI}$$
   vi. $$\Delta_{2L} = \frac{1}{2} \times \frac{320}{EI} \times 12 \times \left(\frac{12 + 4}{3}\right) \times \frac{1}{12} + \frac{1}{2}\left(\frac{2}{3}\right) \times \frac{360}{EI} \times 12 = \frac{2293.33}{EI}$$
5. **To Get Flexibility Matrix :**
   i. Unit force is applied in determinate structure in the coordinate direction 1 as shown in Fig. 20(a). Conjugated beams and loading on them are as shown in Fig. 20(b).



(a) Unit load in coordinate direction 1.



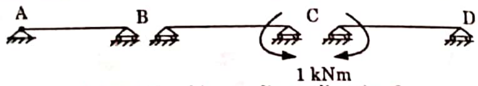
(*b*) Conjugate beam and loading.

Fig. 20.

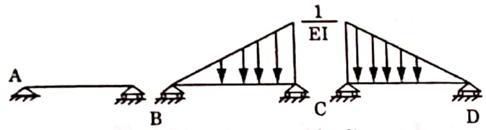
δ_{11} = (2/3) × 12/(2*EI*) + (2/3) × 12/(2*EI*) = 8/*EI*

and δ_{11} = (1/3) × 12/(2*EI*) = 2/*EI*

ii. Unit force is applied in coordinate direction 2 as shown in Fig. 21(*a*). The conjugate beam and loading on it are as shown in Fig. 21(*b*).



(*a*) Unit Lead in coordinate direction 2.



(*b*) Conjugate beam and loading.

Fig. 21.

δ_{12} = (1/3) × 12/(2*EI*) = 2/*EI*

and δ_{22} = (2/3) × 12/(2*EI*) + (2/3) × 12/(2*EI*) = 8/*EI*

6. The final displacements at 1 and 2 are zero.
7. Hence, the flexibility equation is given by, [δ] [*P*] = [Δ] – [Δ_L]

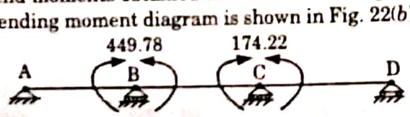
$$\begin{bmatrix} 8/EI & 2/EI \\ 2/EI & 8/EI \end{bmatrix}\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 3946.67/EI \\ 2293.33/EI \end{bmatrix}$$

$$\frac{1}{EI}\begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = -\frac{1}{EI}\begin{bmatrix} 3946.67 \\ 2293.33 \end{bmatrix}$$

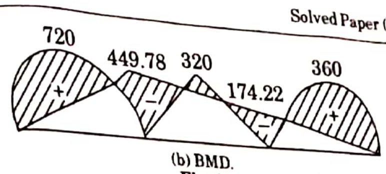
$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = -\begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix}^{-1}\begin{bmatrix} 3946.67 \\ 2293.33 \end{bmatrix}$$

$$= -\frac{1}{(64-4)}\begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix}\begin{bmatrix} 3946.67 \\ 2293.33 \end{bmatrix} = \begin{bmatrix} -449.78 \\ -174.22 \end{bmatrix}$$

8. The end moments obtained are indicated in Fig. 22(*a*) and hence the bending moment diagram is shown in Fig. 22(*b*).



(*a*) End moments.



(b) BMD.

Fig. 22.

b. Analyse the continuous beam shown in Fig. 23, by stiffness matrix method. Also sketch the bending moment diagram.

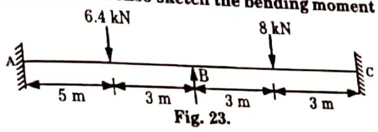


Fig. 23.

Ans.

1. Degree of Kinematic Indeterminacy and Selection of Coordinate :

Since here unknown displacement happens at *B* only. Hence the kinematic indeterminacy of the given beam is 1. Hence select a coordinate at *B* only as shown in Fig. 24.

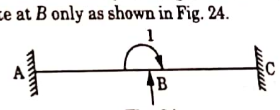


Fig. 24.

2. Fully Restrained Structure and Forces Development in the Coordinate Direction :

i. Fully restrained structure to be obtained by imposing the restraints of all joints as shown in Fig. 25.

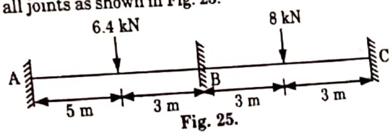


Fig. 25.

ii. Fixed end Moments :

$M^F_{AB} = -Wab^2/L^2 = -6.4 \times 5 \times 3^2/8^2 = -4.5$ kN-m

$M^F_{BA} = Wa^2b/L^2 = 6.4 \times 5^2 \times 3/8^2 = 7.5$ kN-m

$M^F_{BC} = -Wab^2/L^2 = -8 \times 3 \times 3^2/6^2 = -6$ kN-m

$M^F_{CB} = +6$ kN-m

iii. The force developed in the coordinate direction,

$P_L = 7.5 - 6 = 1.5$ kN-m

3. Stiffness Matrix :

i. Since there is only one coordinate. Hence the size of the stiffness matrix [*K*] will be of the order 1 × 1.

ii. To obtain stiffness matrix [K], apply unit displacement in the coordinate direction as shown in Fig. 26.

iii. Force developed in the coordinate direction to be obtained by using slope deflection equation.

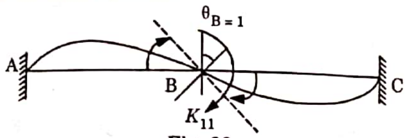


Fig. 26.

$$K = (2EI/8)(\theta_A + 2\theta_B - \delta) + (2EI/8)(2\theta_B + \theta_C - \delta)$$
$$= (2EI/8)(0 + 2 \times 1 - 0) + (2EI/8)(2 \times 1 + 0 - 0)$$
$$= (EI/4) \times 2 + (EI/3) \times 2 = 7EI/6$$

Hence the stiffness matrix, $[K] = [7EI/6]$

4. Final Forces :

Final force acting in coordinate direction is zero

i.e., $[P] = [0]$

5. Stiffness Equation :

i. The stiffness equation is given by,

$$[K][\Delta] = [P - P_L]$$
$$[7EI/6][\Delta] = [0 - 1.5]$$
$$[\Delta] = (6/7EI) \times [1]^{-1}[-1.5] = -9/(7EI)$$

ii. $\theta_B = -9/(7EI)$

iii. $\theta_A = \theta_C = 0$ ($\because$ *A* and *C* are fixed end)

6. Final End Moments :

$M_{AB} = -4.5 + (2EI/8)[-9/(7EI)] = -4.82$ kN-m

$M_{BA} = 7.5 + (2EI/8)[-2 \times 9/(7EI)] = 6.857$ kN-m

$M_{BC} = -6 + (2EI/6)[-2 \times 9/(7EI) + 0] = -6.857$ kN-m

$M_{CB} = 6 + (2EI/6)[-9/(7EI) + 0] = 5.57$ kN-m

7. Simply Supported Bending Moment :

i. Moment in span $AB = Wab/L = 6.4 \times 5 \times 3/8 = 12.0$ kN-m

ii. Moment in span $BC = WL/4 = 8 \times 6/4 = 12.0$ kN-m

8. Bending Moment Diagram :

The bending moment diagram is drawn on tension side as shown in Fig. 27.

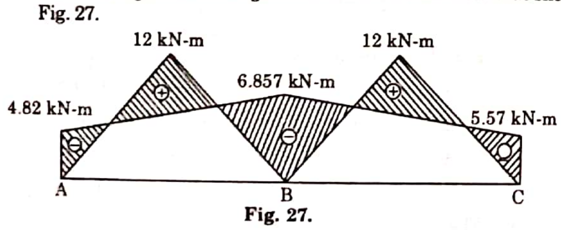


Fig. 27.

7. Attempt any one part of the following : **(7 × 1 = 7)**

a. Find the collapse load for the frame shown in Fig. 28.

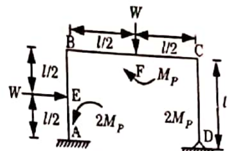


Fig. 28.

Ans.

1. Degree of static indeterminacy,

   r = 3 (Number of closed loops) – Number of releases = 3(1) – 1 = 2

   (Since the end '*D*' is not restricted against rotation,

   $\because$ Number of release = 1)

2. Number of possible plastic hinges, $N = 5$ (*i.e.*, at *A*, *B*, *C*, *D* and *F*)
3. Number of independent mechanisms, $n = N - r = 5 - 2 = 3$
4. These three numbers of independent mechanisms are :

i. Beam mechanism.

ii. Column mechanism.

iii. Panel mechanism or sway mechanism.

In addition to above three mechanisms, there will be the combined mechanisms.

i. Beam Mechanism :

a. Beam mechanism is shown in Fig. 29.

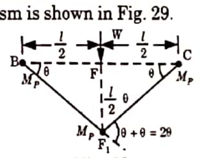


Fig. 29.

b. External work done, $W_E = W \times FF_1 = W \times (l/2)\,\theta$

c. Internal work done, $W_I = M_p\theta + M_p \times 2\theta + M_p\theta = 4M_p\theta$

d. Equating external work done and internal work done, we get

$$Wl\theta/2 = 4M_p\theta \quad \text{...(1)}$$
$$\therefore \quad W_c = 8M_p/l$$

ii. Column Mechanism :

a. In the column mechanism plastic hinges are formed at *A*, *B* and *E* as shown in Fig. 30.

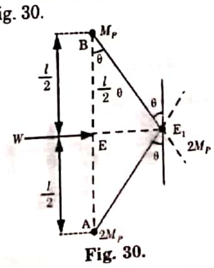


Fig. 30.

b. External work done, $W_E = Wl\theta/2$

c. Internal work done, $W_1 = 2\,M_p\theta + 2\,M_p(\theta + \theta) + M_p\theta = 7\,M_p\theta$

d. Equating external and internal work done

$$Wl\theta/2 = 7\,M_p\theta$$

$$W_c = 14M_p/l \quad ...(2)$$

iii. Panel Mechanism :

a. In this mechanism plastic hinges are formed at *A*, *B* and *C* only.

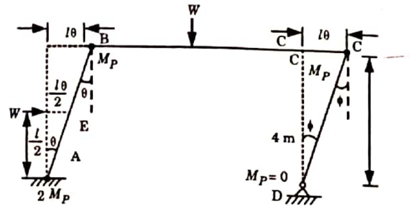


Fig. 31.

b. External work done, W_E = Load × Displacement at *E*

$$= W \times EE' = W \times (l/2) \times \theta = Wl\theta/2$$

c. Internal work done, $W_I = 2\,M_p\theta + M_p\theta + M_p\theta = 4\,M_p\theta$

d. Equating external and internal work done

$$Wl\theta/2 = 4\,M_p\theta$$

$$W_c = 8M_p/l \quad ...(3)$$

iv. Combined Mechanism :

a. This mechanism is shown in Fig. 32.

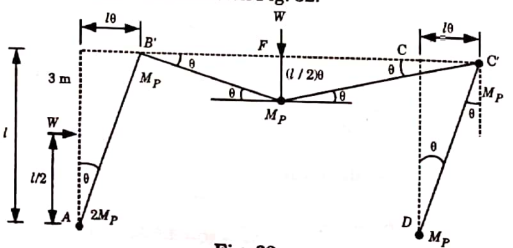


Fig. 32.

The plastic hinges are formed at *A*, *F* and *C*.

b. External work done, $W_E = W \times \frac{l}{2}\theta + W \times \frac{l}{2}\theta = \frac{Wl\theta}{2} + \frac{Wl\theta}{2} = Wl\theta$

c. Internal work done,

$$W_I = 2\,M_p\theta + M_p(\theta + \theta) + M_p \times (\theta + \theta) = 6\,M_p\theta$$

d. Equating external and internal work done

$$Wl\theta = 6\,M_p\theta$$

$$W = 6M_p/l \quad ...(4)$$

From eq. (1), (2), (3) and (4) we consider least value of

Hence collapse load, $W_C = 6M_p/l$

b. Analyse the propped cantilever beam loaded as shown in Fig. 33 and determine the collapse load.

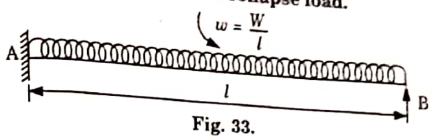


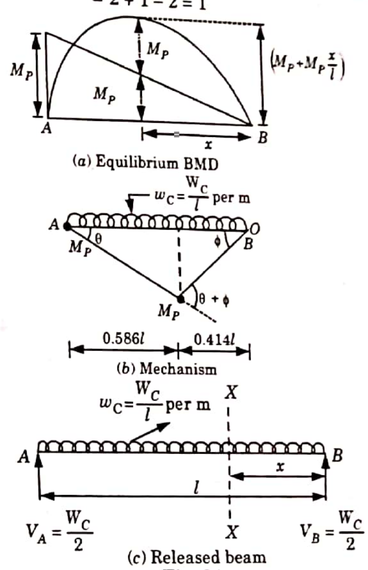
Fig. 33.

Ans.

1. Degree of static indeterminacy,

$$r = \text{Vertical reactions} + \text{End moments} - 2$$

$$= 2 + 1 - 2 = 1$$



(*a*) Equilibrium BMD

(*b*) Mechanism

(*c*) Released beam

Fig. 34.

2. Number of possible plastic hinges, $N = 2$ (at *A* and in *AB*)
3. Number of independent mechanism, $n = N - r = 2 - 1 = 1$
4. Consider the moment at *A* as redundant and that is reaches M_P.
5. The second hinge will be form where the net positive bending moment is maximum.
6. The equilibrium bending moment diagram, mechanism and simply supported beam are shown in Fig. 34, (a), (b) and (c) respectively.
7. Bending moment at *X*.

$$M_X = \frac{W_c}{2}x - \frac{W_c}{l}x\frac{x}{2}$$

8. From Fig. 34(a) and 34 (b), we get

$$M_p + M_p \frac{x}{l} = \frac{W_c}{2} x - \frac{W_c x^2}{2l}$$

$$M_p\left[1 + \frac{x}{l}\right] = \frac{W_c x}{2} - \frac{W_c x^2}{2l}$$

$$M_p\left[\frac{l+x}{l}\right] = \frac{W_c}{2} x\left(1 - \frac{x}{l}\right) = \frac{W_c}{2} x\left(\frac{l-x}{l}\right)$$

$$M_P = \frac{W_c}{2} x \frac{(l-x)}{(l+x)} = \frac{W_c}{2}\left[\frac{(lx - x^2)}{(l+x)}\right]$$

9. For M_P to be maximum; $dM_p/dx = 0$

Now, $$\frac{dM_p}{dx} = \frac{d}{dx}\left[\frac{W_c}{2}\left\{\frac{(lx - x^2)}{(l+x)}\right\}\right] = 0$$

$$= \frac{W_c}{2}\left[\frac{(l+x)(l-2x) - (lx - x^2)(1)}{(l+x)^2}\right] = 0$$

$$W_c/2 \neq 0$$

$$(l + x)(l - 2x) - (lx - x^2) = 0$$

$$l^2 - 2lx + lx - 2x^2 - lx + x^2 = 0$$

$$l^2 - 2lx - x^2 = 0$$

$$x^2 + 2lx - l^2 = 0$$

$$x = \frac{-2l \pm \sqrt{(2l)^2 - 4 \times l \times (-l^2)}}{2 \times 1} = \frac{-2l \pm \sqrt{4l^2 + 4l^2}}{2}$$

$$= \frac{-2l \pm \sqrt{8l^2}}{2} = \frac{-2l \pm 2.828l}{2}$$

Taking + ve value $x = \frac{-2l + 2.828l}{2} = \frac{0.828l}{2} = 0.414\, l$

10. From mechanism Fig. 34 (b),

$$0.586l\theta = 0.414\, l\, \phi$$

$$\phi = \frac{0.586}{0.414} = 1.4155\theta$$

$$\theta + \phi = \theta + 1.4155\theta = 2.4155\theta$$

11. External work done, $W_E = \frac{W_c}{l} \times \frac{1}{2} \times l \times 0.586l\theta = 0.293\, W_c l\theta$

12. Internal work done, $W_I = M_P\theta + M_P(\theta + \phi) + 0 = 3.4155\, M_p\theta$

13. Equating external and internal work done, we get

$$0.293\, W_c l\theta = 3.4155\, M_P\theta$$

Collapse load, $W_c = \frac{3.4155\, M_P}{0.293l} = \frac{11.657\, M_P}{l}$

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